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A Sound Art Installation based on the theory of coupled nonlinear oscillators

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ABSTRACT

The objectives of this master thesis were to create a sound art installation that: (I) conceptually functions as a metaphor for social behavior and (II) explore the concept of entrainment as a basis of creating rhythm-based abstract music.

The ambition was to create a system in which sounds are being influenced by surrounding sounds, similar to how people get influenced by surrounding people and thereby adjust their behavior to the surroundings.

The sounds are controlled to have a tendency to synchronize with each other, but never lock to a common frequency perpetually. Instead, the sounds are influenced by each other in order to create abstract rhythms that are distinctly characterized by the nature of coupled oscillators. Thereby, I can explore as well as present new forms of musical expression.

The theory of coupled oscillators is applied to control the sounds; specifically the Kuramoto model has been appointed to be a suitable representation for the interaction between individuals and is thus used to control the interaction between the different sounds.

A sound art installation, consisting of a room with four speakers and one projector, was created. A set of 256 sounds was used, each representing an oscillator (or individual). The sounds are being influenced by their surrounding sounds to a certain degree and thereby creating rhythmic patterns that are played back through the speaker system, using a panning function that divides all sounds between the speakers and thereby positions each sound in the room according to each sounds' logic position in space. A visual representation of the behavior of the sounds is presented on one of the walls in the room.

Keywords: sound, art, music, coupled oscillators, synchronization, rhythm.

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I'd like to thank my instructor Dr. Mats Nordahl for his stimulating and valuable comments and advices along the way. His understanding and knowledge of the field have provided substantial contributions to this project.

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Introduction: Artistic objectives and framework

The overall idea of this project is to make a sound art installation which is based on the theory of coupled nonlinear oscillators. An essential inspiration to this project is the nature of group behavior, how individuals get influenced by the other individuals in their surroundings. Phenomenon of this kind, how individuals can get *synchronized* to the other individuals in the surroundings, can be explained by models of coupled oscillators. Phenomenons of this kind are for example frogs that synchronize their singing to surrounding frogs, and crickets that chirp in unison.

The artistic objective of the project is to create a sound art installation that is based on the nature of coupled oscillators; to let a set of sounds influence each other, in a way similar to how frogs synchronize their singing to each other, or how people get influenced by their surroundings.

There are many artists that have used the *sounds* of, for example, synchronized frogs and crickets in their works, such as the avant-garde composer and electronic musician Pauline Oliveros[1] who is deeply influenced by the sound of frogs[2] and which is reflected in the music she makes (for example on Alien Bog[3]). Oliveros, who is referred to as a pioneer of the concept of deep listening, has elements of sounds of frogs and crickets in the music she makes. Similarly to Oliveros, the sound artist Marc Behrens[4] also uses frogs and insects as sources for his compositions[5]. On the contrary to Oliveros' and Behrens' work, which includes recordings of synchronized singing frogs, the work in this thesis deals with the synchronization phenomenon in itself rather than the sound generated from a frog pond or a field of crickets.

The sounds in this installation are thus more of a representation of the relation between individuals, how also people get influenced by others. The work by the artist and field biologist Henrik Håkansson[6] has also often a focus on the relation between the humans, the animals, the insects and the plants, and is also very technology based and often close to scientific experiments. Though, the objective for this work is to study the phenomenon of individuals' influence on each other, more than the relations as in Håkansson's case. People's influence on each other is also the subject in Norwegian Dag Svanæs'[7] work Pulse: Wearable Jewellery, where blinking jewellery worn by different persons adjusts their blinking to confronting people's worn jewellery. Similarly to how the individual pieces of jewellery in Svanæs' work influence the jewellery on confronting people, the sounds in this installation gets influenced by surrounding sounds.

This sound art installation will thus be based on mathematical models for coupled nonlinear oscillators. The sounds used in the installation will each be influenced by the surrounding sounds, just as individuals get influenced by other individuals in real life. Thus, apart from being strictly a sound art installation, the installation can also be seen as a metaphor for social behavior.

Social behavior and group dynamics

In many aspects humans appear to be not that different from many other species that synchronize their behavior to the other individuals in the surroundings. Humans seem to adjust their behavior to other people in their surroundings; in a way similar to how fireflies adjust their illumination to other fireflies' illumination, or the way frogs adjust their singing to other frogs' singing. For example, it is very common that an audiences applauses falls into sync after a while, to a certain extent. The people in the audience spontaneously adjust their clapping to the others' "frequency" of the clapping. Each person have their own "natural frequency" of their clapping, a frequency that they start their clapping in, but after a while the surrounding applauses' frequencies influences each individuals clapping and synchronizations evolves. Parts of the audience then start to clap in a common tempo.

Another example might be the consequence of a person watching another person that starts to yawn. Then it is very likely that the observing person also starts to yawn. Or, what can happens at a party when some people suddenly stop talking, then it might be that *everybody* consequently stops talking and it gets all quiet. Similarly, people tend to adjust to other people's way of dressing, and fashion trends spreads. Suddenly one clothing brand can go from not being sold at all to a state where millions of pieces are sold all over the world. An example is the Hush Puppies shoes, which barely sold any pairs at all in 1994 and the model were about to be discontinued when a few club kids in the East Village in New York suddenly started to buy Hush Puppies in some cheap stores. A couple of Manhattan designers then used these shoes for a couple of shows, and within a couple of years the shoe were sold in every mall throughout America and in stores all over the world. Thereby, fashion trends can evolve and spread rapidly as a result to people's social behavior. People get influenced by some people in their surroundings and to a certain extent behave in a similar way. Malcolm Gladwell discusses in his book *The Tipping Point – how little things can make a big difference*[8] how human behaviour crosses thresholds and "tips", i.e. how humans get influenced by their neighbours and causes a spreading tendency throughout our society.

Individuals' social behavior, and individuals' tendency to adjust their behavior to other individuals' behavior, seems to be a phenomenon that exist in many levels. Looking at the earth as a whole, people seem to gather in clusters. Everybody wants to be where everybody else is, doing what other people do. More and more people are moving to the bigger cities around the world - major cities that act like nodes which attract more and more people to move there. In the cities there are certain paths that are popular, where everybody wants to be. Some streets that people want to walk. Why is it that a massive amount of people choose to gather at that one street? Or in a wider perspective, why do they gather in a big city? Similarly, why is it that some restaurants are always fully booked? And, why do people want to wear clothes that are "trendy"? Or why are they trendy? Likewise, how could the Nazi leaders get millions of people to participate in the Holocaust? Or why is it that the stock market exhibit occasional large fluctuations that cannot be traced to any correspondingly significant piece of information, which results in booms and crashes in the market?

There are numerous examples on the way people adopt their behavior to their surroundings, as a consequence of group pressure or powers of authority, etc. The following subchapters discuss some examples of this character.

Conformity

Solomon Asch did a study of conformity in 1958, known as the Solomon Asch experiment[9], where he let a number of college students participate in an experiment and scheduled them to participate one at a time. When each subject arrived they were seated at a table together with seven to nine other people who were supposed to be subjects as well but were in fact associates with the experimenter and their behavior had been well scripted. Asch told them that they should participate in a study on visual perception. Two cards were placed on the table, one showing a vertical line and the other is showing three vertical lines of varying length. The experimenter asked the people around the table, one by one, which line on the second card that has the same length as the first card. The procedure was repeated with a total of 18 sets of bars. The confederates were told to give false answers on 12 of the 18 trials and the only real subject were arranged to be the second last person to announce his answer. The question was really simple and the answer was obvious. The cards were just as in the figure below and the subject was to say which two lines have the same length.

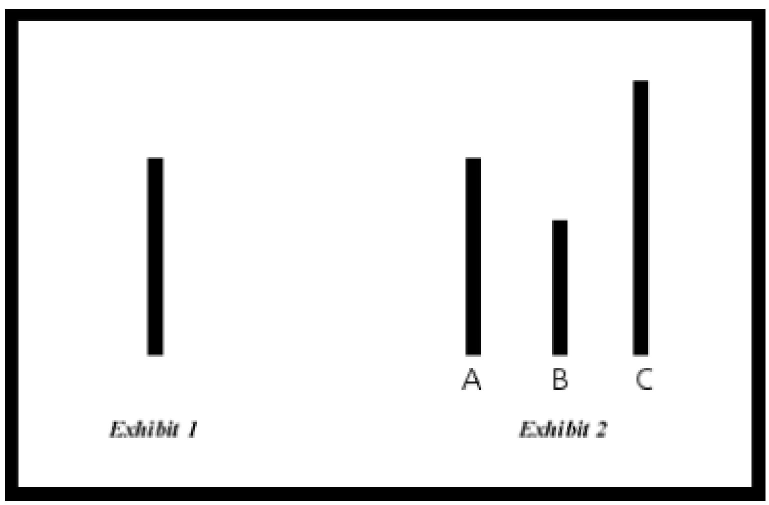


Figure 1: Illustration of the Solomon Asch Experiment

The confederates announced their incorrect answers aloud one by one and by the end of the round the real subject to a great extent also announced the same incorrect answer as a consequence of the social pressure. Out of 50 subjects, 37 conformed to the majority at least once, and 14 of them conformed on more than 6 of the 12 trials. When the subject was faced with a unanimous incorrect answer by the group members, the mean subject conformed on one third of the trials. Asch was shocked by the results: “The tendency to conformity in our society is so strong that reasonably intelligent and well-meaning young people are willing to call white black. This is a matter of concern. It raises questions about our ways of education and about the values that guide our conduct."

Asch's experiment on the question to which extent pressure from other people affects us, and its following questions to the subjects, resulted in Asch's conclusion: "Apparently, people conform for two main reasons: because they want to be liked by the group and because they believe the group is better informed than they are."

Asch also found that one of the situational factors that influence conformity is the size of the opposing majority. He made additional studies where he varied the number of confederates, which resulted in that fewer subjects followed the opposing majority if they had another person thinking the same way as themselves. The studies showed that it is difficult to be a minority of one but not so difficult to be part of a minority of two. Therefore the group size and the cohesiveness of the group's opinion are parameters for the degree of conformity. Asch concluded that it is difficult to maintain that you see something when no one else does. Asch found that "the group pressure implied by the expressed opinion of other people can lead to modification and distortion that effectively can make you see almost anything".

Another example on the way people conform to their current situation and surroundings is the Stanford Prison Experiment[10].

The Stanford Prison Experiment resulted in a videotaped demonstration of how ordinary people middle-class college students can do things they would have never believed they were capable of doing.

On Sunday morning, August 17, 1971, nine young men were "arrested" in their homes by the Palo Alto police. The arrestees were among about 70 young men, mostly college students willing to earn $15 a day for two weeks, by participating as volunteers for an experiment on prison life, which had been advertised in the *Palo Alto Times*.

After interviews and a series of psychological tests, the two dozen that were judged to be the most normal, average and healthy were selected to participate, assigned randomly either to be guards or prisoners. Those who would be prisoners were booked at a real jail, then blindfolded and driven to campus where they were led into a makeshift prison in a basement.

Those assigned to be guards were given uniforms and instructed that they were not to use violence but that their job was to maintain control of the prison.

From the perspective of the researchers, the experiment became exciting on day two when the prisoners staged a revolt. Once the guards had crushed the rebellion, "they steadily increased their coercive aggression tactics, humiliation and dehumanization of the prisoners," Philip Zimbardo, researcher and leader of the experiment, recalls. "The staff had to frequently remind the guards to refrain from such tactics," he said, and the worst instances of abuse occurred in the middle of the night when the guards thought the staff was not watching. The guards' treatment of the prisoners included such things as forcing them to clean out toilet bowls with their bare hands and act out degrading scenarios, or urging them to become snitches that resulted in extreme stress reactions that forced the researchers to release five prisoners, one a day, prematurely.

The prisoners' rebellion also played an important role in producing greater solidarity among the guards. After the revolt, it was no longer just an experiment, no longer a simple simulation. Instead, the guards saw the prisoners as troublemakers who were out to get them, who might really cause them some harm. In response to this threat, the guards began stepping up their control, surveillance, and aggression.

Push-ups were a common form of physical punishment imposed by the guards to punish infractions of the rules or displays of improper attitudes toward the guards or institution. Push-ups can be seen as a rather juvenile and minimal form of punishment, though it has later shown that push-ups were also often used as a form of punishment in Nazi concentration camps. One of the guards in this experiment also stepped on the prisoners' backs while they did push-ups, or made other prisoners sit or step on the backs of fellow prisoners doing their push-ups.

Christina Maslach, a recent Stanford Ph.D., had agreed to do subject interviews and came down to the “prison” to familiarize her with the experiment. At first, she said, she found it "dull and boring." "I looked at the prison yard from the point of view of the video camera [that had been set up to monitor it] and there was not much happening. So I went around to the other end of the hall where some guards were waiting to start their next shift." There, she had a pleasant conversation with a "charming, funny, smart" young man waiting to start his guard shift. Other researchers had told her there was a particularly sadistic guard, whom both prisoners and other guards had nicknamed John Wayne. Later, when she looked at the monitor of the prison yard again, she asked someone to point out John Wayne and was shocked to discover it was the young man she had talked with earlier.

"This man had been transformed. He was talking in a different accent a Southern accent, which I hadn't recalled at all. He moved differently, and the way he talked was different, not just in the accent, but in the way he was interacting with the prisoners. It was like seeing Jekyll and Hyde…It really took my breath away."

The study was ended permanently on the sixth day for two reasons. First, it had been learned through videotapes that the guards were escalating their abuse of prisoners in the middle of the night when they thought no researchers were watching and the experiment was "off." Their boredom had driven them to ever more pornographic and degrading abuse of the prisoners.

Second, Christina Maslach strongly objected when she saw the prisoners being marched on a toilet run, bags over their heads, legs chained together, and hands on each other's shoulders. Once she countered the power of the situation, it became clear that the study should be ended. And so, after only six days, the planned two-week prison simulation was called off.

The Stanford Prison Experiment research project showed how people, when put in a new situation, soon changes its personality and behaves in a different way. A recent example of what can happen in the experiment in real life is the human rights abuses that occurred at the Abu Ghraib prison under the authority of American armed forces in the aftermath of the 2003 Iraq war. There, soldiers were thrust into the role of prison guards and began to sadistically torment prisoners there and at other detention sites in Afghanistan and Iraq. Many of the specific acts of humiliation were similar to those that occurred in the Stanford Prison Experiment, according to Zimbardo[11].

Zimbardo’s experiment also has many resemblances to psychologist Stanley Milgram’s studies on the Obedience to Authority, as described in the following chapter.

Obedience

After the World War 2 the world was chocked by what came out of the Eichmann Trials; Eichmann, a high ranking officer in the Nazi Party, was on trial for war crimes and crimes against humanity. The question is: Is it possible that Eichmann and his million accomplices in the Holocaust were just following order? Could we call them all accomplices? Did they just do what the others in their surroundings did, what the people in their surroundings told them to do?

Psychologist Stanley Milgram did a series of studies on the Obedience to Authority, at Yale University in 1961-1962, referred to as The Milgram Experiment[12]. The study focused on the conflict between obedience to authority and personal conscience. He examined justifications for acts of genocide offered by those accused at the World War II, Nuremberg War Criminal trials. Their defense often was based on "obedience" - that they were just following orders of their superiors.

In the experiment, so-called "teachers", who were actually the unknowing subjects of the experiment, were recruited by Milgram through a newspaper ad. The ad said that it was a Psychology experiment investigating memory and learning. The recruited person turned up and was introduced to a stern looking experimenter in a white coat and a rather pleasant and friendly co-subject. The experimenter explained that the experiment would look into the role of punishment in learning, and it was (after lots of drawing to determine roles) decided that the person that answered the ad would be the “teacher” and the other person to be the “learner”. The “learner” was taken to a room where he was strapped in a chair to prevent movement and an electrode was placed on his arm. The "teacher" was taken to an adjoining room which contained a generator. The "teacher" was instructed to read a list of two word pairs and ask the "learner" to read them back. If the "learner" got the answer correct, then they moved on to the next word.

If the answer was incorrect, the teacher was asked to administer an electric shock to the “learner”, starting at 15 volts. For each incorrect answer, the “teacher” was supposed to increase the intensity to the "learner" by 15 volts at a time.

The story given to the "teacher" was that the experiment was exploring effects of punishment (for incorrect responses) on learning behavior. The "teacher" was not aware that the "learner" in the study was actually an actor - merely indicating discomfort as the "teacher" increased the electric shocks.

The generator had 30 switches in 15 volt increments; each was labeled with a voltage ranging from 15 up to 450 volts. Each switch also had a rating, ranging from "slight shock" to "danger: severe shock". The "teacher" was supposed to automatically increase the shock one step each time the "learner" misses a word in the list. Although the “learner” was an actor that were never harmed, the “teacher” *believed* that he was given the “learner” real electric shocks. At some times the “teachers” got worried and questioned the experimenter whether he should continue to increase the shocks and asked who was responsible for any harmful effects, but was then verbally encouraged to continue by the experimenter and were told that the experimenter assumed full responsibility, and the “teachers” then accepted the response and continued the shocking even if some of the “teachers” were extremely uncomfortable in doing so.

65 percent of the “teachers” obeyed the orders to punish the learner to the very end of the 450-volt scale. No subject stopped before reaching 300 volts.

Milgram’s results revealed the theory that only the most severe monsters on the sadistic fringe of society would submit such cruelty. He found that "two-thirds of this studies participants fall into the category of ‘obedient' subjects, and that they represent ordinary people drawn from the working, managerial, and professional classes (Obedience to Authority)."

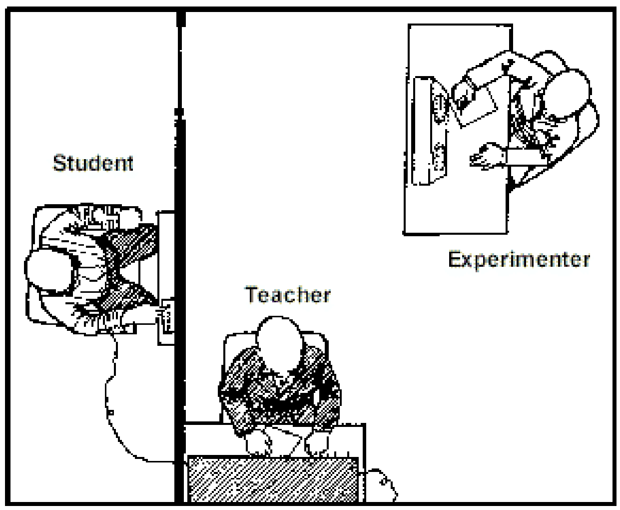


Figure 2: The Milgram Experiment

The experiments and studies described above are examples on related research that has been made on the nature of people’s tendency to adopt their behavior to their current situation and surroundings. Another field of studies that relate to the way a group of individuals (or *oscillators*) influence each other is the studies of entrainment.

Entrainment

The concept of entrainment describes the phenomenon of two or more independent rhythmic processes that interact with each other in such a way that they adjust towards each other and eventually lock in to a common phase and/or periodicity – they get *synchronized*.

Entrainment as a concept was first identified by Christiaan Huygens, who is described in the next chapter, in 1665 and has been applied in various disciplines, from mathematics, physics and biology to linguistics, psychology and social sciences, as well as in the field of music.

There are two basic components involved in all instances of entrainment: there must be two or more autonomous rhythmic processes or oscillators, and the oscillators must interact. The tendency for rhythmic oscillators to adjust in order to match other rhythms has been described in a wide range of systems and over a wide range of periodicities: from fireflies that illuminates in synchrony, through humans adjusting their speech rhythms to match each other in conversation, to sleep-wake cycles synchronizing to the changes of dark and light during the day. Examples has been claimed from the fast frequency oscillators of brain waves to periods extending over many years, and in organisms from the simplest to the most complex. The concept of entrainment describes the coordination of events through interaction, for example the foot tapping that possibly emerges when a person listens to a song. There are many naturally occurring rhythms within the human body; for example the heart beat, blood circulation, respiration, locomotion, eye blinking, secretion of hormones, female menstrual cycles, and many others. It has been suggested that all human movements are inherently rhythmic. For example Mari Jones writes that "all human performance can be evaluated within a rhythmic framework[13]" and Bernieri, Reznick and Rosenthal write that "human behavior is understood to occur rhythmically and therefore can be described in terms of cycles, periods, frequencies, and amplitudes[14]".

A lot of recent research has concentrated upon studying the nature of rhythmic processes in living organisms. Steven Strogatz and others states that biological or psychological rhythms appear to be essential to life itself[15] while some has gone so far as to characterize any organism as a loosely coupled population of oscillators[16].

Allen C. Bluedorn explains the entrainment process as "the process in which the rhythms displayed by two or more phenomena become synchronized, with one rhythm often being more powerful or dominant and capturing the rhythm of the other. This does not mean, however, that the rhythmic patterns will coincide or overlap exactly; instead, it means the patterns will maintain a *consistent relationship* with each other"[17].

As in Bluedorn's definition of the process of entrainment, it is the aim of this sound art installation project to let a set of sounds to have a tendency to synchronize with each other, but never lock to a common frequency eternally. Instead, the sounds will be influenced by each other to a certain limited degree, as is further described in the following chapters.

Sound as metaphor

Peoples' surroundings obviously have a lot of impact on their behavior. As the examples in the previous chapter illustrates, people's tendency to conform to a crowd can in some situations lead to very unpleasant results. Peoples tendency to adopt their behavior to their surroundings is very evident in many other situations as well; people appear to, to a great extent, listen to their neighbors and get influenced by what everybody else does, to synchronize their behavior to what surrounding people do and thereby behave, consume, etc. in a similar way.

In a very short timeframe a major group of people can change the way they dress, or the music they listen to, just like a flock of birds changing direction. "If they do it, it must be right, I do it too", as the persons participating in Solomon Asch's experiment thought. Or if some specific people say that it is right, then it must be the way to go. The subjects in Solomon Asch's experiment said they did as the majority of the people even if they doubted that the rest of the people were right, as they believed that the other people were better informed than themselves. What is it that makes people dress a certain way – do they believe that the fashion designers or shop owners are better informed, or the other people around them? Or do they simply want to fit in to a common pattern, to a trend? And what is it that makes one album of an unknown debut artist sell millions of copies of the first album in just a few weeks? And similarly, why does a debut book of an unknown author turn into a bestseller?

One little fashion trend on the other side of the globe can rapidly spread to the other side of the world. The work Coca-Locust Family[18] by artist Thomas Broomé, consisting of a lot of locusts (a kind of crickets) made out of Coca Cola cans is an illustration of how everybody tends to drink Coca Cola, and like crickets, or locusts, these cans spreads over the world. Similar to Broomé's locusts, the sounds in this work can be seen as a metaphor for how social phenomenon spreads throughout our society.

The way individuals get influenced by surrounding individuals, and how synchronizations in a group of individuals can evolve, could as earlier said be explained by mathematical models of coupled nonlinear oscillators, which is used as a basis for this sound art piece. This sound installation is consequently intended to be seen as a metaphor for how people tend to adjust their behavior to their surroundings. However, the intention is also to present new musical expressions, based on the phenomenon of coupled oscillators.

Coupled oscillators as musical expression

A major part of this project consists of studies of the nature of coupled oscillators, and how these mathematical models can be used for musical expression.

By using the phenomenon of coupled oscillators as a foundation in a musical piece, new ways for creating rhythms are established. By letting a set of sounds, each sound representing an oscillator, be coupled to each other to a certain specified degree, it is possible to create rhythms and sounds that are strongly characterized by the nature of coupled oscillators explicitly, and that could not be achieved in any other way.

The theory of coupled oscillators can thus be used as a base for controlling a sound art installation that is built on rhythm, and thereby create musical expressions that are based on rhythm. By letting different sounds influence each other and in that way creating rhythmic patterns that are built on individual sounds that to some degree becomes synchronized, the musical expression becomes a rhythmic structure that is consistently evolving, built on the patterns of the individual sounds that throughout maintain a consistent relationship to each other, just as in Allen C. Bluedorn's definition of the process of entrainment.

Australian mathematician and composer Gordon Monro's music piece *Peer Pressure*[19], relates to the idea of this project: Monro simulates a group of 200 fireflies that flash in synchrony, using buzz from a cicada to represent each firefly. Ph.D. Jonathan Bachrach has also done a drum ensemble, *Beatrix*[20], at MIT, that is built on the composition of multiple drum patterns where Bachrach also uses temporal synchronizations between different drum patterns in order to create model of an African drum ensemble.

In reminiscence of Christiaan Huygens discovery of the phenomenon of entrainment where he studied two ticking pendulum clocks, a relating musical piece is also the Hungarian composer György Ligeti's modernistic and abstract music piece 'Poème Symphonique for 100 metronomes'[21], composed in 1962. Ligeti's composition is performed by a concert of 100 metronomes, where each metronome is wound to its maximum extent and then left to tick down. The sound is at first a static crackle, but after a few minutes some patterns can be noticed in the sound and individual metronomes can be more clearly made out. The piece always ends with one final metronome that ticks alone for a few beats.

Similar to 'Poème Symphonique for 100 metronomes' is the art installation 'On Potent Impotence'[22] created in 1989 by Czech artist Ivan Kafka, which displays 20 identical clocks (also exhibited with metronomes instead of clocks) that sits on 20 pedestals. Kafka's installation also creates rhythms from the ticks that each clock produces each second, and can thereby be experienced as a musical piece similar to Ligeti's metronome piece (even though Kafka's intention of his installation was to metaphorically suggest the potential for revolution in the masses, both in the classic Marxist sense, and also the actual overthrow of the totalitarian regimes that was occurring at the time the work was made).

One can ask the question if the ticks of the individual metronomes in Ligeti's composition, and the clocks in Kafka's installation, influence each other in terms of entrainment. Whether they affect each other to some degree or not would apparently depend on their common support or possibly air movements. James Pantaleone at University of Alaska has made a study on synchronization of metronomes[23], where the metronomes were placed on a freely moving common base, and Pantaleone then found that the metronomes soon locks in to an in-phase synchronization and with anti-phase synchronization occurring under special conditions. Though, Pantaleone's experiments had better conditions for entrainment as the metronomes were placed next to each other on a freely moving common support, compared to Kafka's installation where the clocks were standing on separated fixed pedestals. If there was any degree of entrainment in Ligeti's and Kafka's pieces, it was probably to a very limited degree.

The intention in this work is to let the concept of entrainment play a lot bigger role in the shaping of the musical expression, compared to the pieces by Ligeti and Kafka.

Specific aims

The specific aims for this sound art installation are to: create a sound installation that conceptually functions as a metaphor for social behavior and at the same time also elaborates with the concept of entrainment as a basis for creating rhythm-based abstract music.

As in Bluedorn's definition of the process of entrainment, it is the aim to let sounds have a tendency to synchronize with each other, but never lock to a common frequency perpetually. Instead, the aim is to let the sounds be influenced by each other to a certain degree, to a degree upon which the sounds clearly influence each other but do not totally fall into synchronization, and thereby essentially create rhythms that provides a musical expression that I personally find interesting, and which also images the phenomenon of social interaction.

The aim for the presentation form of the installation is to represent each oscillator (or individual) with a sound, a sound which is mapped to a physical location in the room and panned between four speakers, one in each corner of the room, in order to distribute the each sound between the speakers according to its logic mapping in the space. The aim is also to make a visual representation of the system that can be projected on one of the walls in the exhibition room.

In order to realize the project with the aims described above, the theory of coupled oscillators and specifically the Kuramoto model is applied, as it has been appointed to be a suitable representation for interaction between individuals, among other things.

Applied Theory

Coupled oscillators

The theory of coupled oscillators was initiated by the observation Christiaan Huygens[24], the inventor of the pendulum clock, made in 1665 when he noticed that his two pendulum clocks tended to sync with each other over time. His two clocks were hanging beside each other swinging in perfect synchrony, and would not fall out of it. When he tried to disturb them they would regain perfect synchrony within half an hour. When he moved one of the clocks to the other side of the room they would not fall into sync. The synchronization was therefore dependent of the distance. Huygens suspected that they influenced each other, perhaps through tiny air movements or vibrations in their common support, when they were close to each other. The observation Huygens made initiated a subgenre of mathematics: the theory of coupled oscillators.

An oscillator is any system that executes periodic behaviour, for example, as above, a swinging pendulum that returns to the same spot in space at regular intervals, and its velocity rises and falls with regularity.

Coupled oscillators can be found throughout in the natural world but they are especially conspicuous in living things: spinal cord that control rhythmic behaviours such as breathing, etc., and also oscillators that are not confined in the same organism: for example crickets that chirp in unison or singing frogs.

Simplified models of coupled oscillators, that retain the essence of their biological prototypes, have been studied by mathematical biologists since around 1960.

A number of people have since then made major contributions to the knowledge of this area.

Related research

Arthur T. Winfree[25] (1942-2002) was one of the real pioneers in biological oscillators. Winfree was known as a scientist working in many disciplines; though he was primarily know for his work on biological rhythms and on applied mathematics. Winfree made extensive work on the field recognized as Collective Synchronization. His first paper[26] concerned the mutual synchronization of biological oscillators. With this paper Winfree was the first person to make mathematical progress on the question: How is it that thousands of neurons or fireflies or crickets, all firing or flashing or chirping at the same time, without any leader or signal from the environment? What Winfree made in his studies which hadn't been done before (by Wiener[27] et al.), was to consider more than one single forced oscillator, or two coupled oscillators, which hadn't been the case in previous research. Winfree considered a gigantic system of oscillators with randomly distributed frequencies. He made a few simplifications to the problem; he argued that amplitude variations and the limit of weak coupling could be neglected and that oscillators could be described solely by their phases along their limit cycles. The work by Winfree – this “phase model” reduction – stimulated later mathematical work by Kuramoto and others.

Steven H. Strogatz[28], professor at the Department of Theoretical and Applied Mechanics at Cornell University, has done extensive studies on areas that relates to this project. Strogatz was the author of the recently published popular science book *Sync: The Emerging Science of Spontaneous Order*[29] which discusses how spontaneous order occurs at every level of the cosmos. Strogatz explores the mysterious synchrony achieved by fireflies that flash in unison by the thousands, crickets that chirp in unison and the question of what makes our own body clocks synchronize with night and day and even with one another. His research is based in his broad interest in applied mathematics and mathematical biology and he has been working on a wide range of projects dealing with coupled oscillators, such as the examples mentioned above. Strogatz also discusses biological rhythms etc. in his previous book *Nonlinear Dynamics and Chaos: With Applications in Physics, Biology, Chemistry, and Engineering (Studies in Nonlinearity)*[30] as well as in a number of other articles and scientific publications.

The Kuramoto model

To realize the implementation of a sound installation based on singing frogs, mathematical models of nonlinear coupled oscillators is, as earlier mentioned, used as a starting point. Strogatz, Ian Stewart[1] and others’ theory and mathematical models are used for representing the behaviour of individuals’ patterns for entrainment. Specifically, the Kuramoto model is used.

The Kuramoto model describes the phenomenon of collective synchronization, in which an enormous system of oscillators spontaneously locks to a common frequency, despite the inevitable differences in the natural frequencies of the individual oscillators. Norbert Wiener was the first to study collective synchronization, but his mathematical formulations turned out unsuccessful. Winfree, mentioned above, continued on Wiener’s work and got more fruitful results, with his “phase model” reduction. Yoshiki Kuramoto began working on collective synchronization in 1975. Based on Winfree’s work he published his first paper on the topic, announcing some exact results about what come to be called the Kuramoto model:

$$\theta_i = \omega_i + \frac{K}{N}\sum_{j=1}^{N}\sin(\theta_j - \theta_i) \quad , \qquad i = 1,\ldots,N$$

In the formula above, θ_i is the phase for oscillator i, ω_i is the natural frequency of oscillator i, K is the coupling factor and N is the number of oscillators.

Instead of just studying an oscillator's behaviour over time, one can also consider the oscillators motion through *phase space*. This is an abstract space, whose coordinates describe the *state* of the system. This is consequently what the Kuramoto model describes.

The algorithm to be used in this artwork is based on the formula above, i.e. the Kuramoto model of coupled nonlinear oscillators.

Examinations and tests

Implementation of the Kuramoto model

A major part of this thesis consists of investigations, examinations and tests related to the behaviour of coupled nonlinear oscillators, and the Kuramoto model. The implementation is made in C++ in the Microsoft Visual Studio .NET environment and run in win32 console application mode.

The Runge-Kutta method is used in order to implement the following formula, referred to as the Kuramoto model, described in the Theory chapter:

$$\theta_i = \omega_i + \frac{K}{N}\sum_{j=1}^{N}\sin(\theta_j - \theta_i) \quad , \qquad i = 1,...,N \; .$$

The Kuramoto model above describes the oscillators' motion in *phase space*, an abstract space whose coordinates describe the *state* of the system. θ_i is the phase for oscillator i, ω_i is the natural frequency of oscillator i, K is the coupling factor and N is the number of oscillators.

This differential equation above is implemented by using the Runge-Kutta method of 4th order. Given start values for $y[i]$ (for all oscillators, $i = 1 \cdots N$), and for their derivatives $\omega[i]$ ($i = 1 \cdots N$), the incremented $y[i]$, below called $y_{new}[i]$, is calculated by the following steps, where $k1$, $k2$, $k3$ and $k4$ are variables for storing temporary data needed to compute y_{new}:

$$k1[i] = h\left(\omega[i] + \frac{K}{N}\sum_{j=1}^{N}\sin(y[j] - y[i])\right) \qquad i = 1 \cdots N$$

$$k2[i] = h\left(\omega[i] + \frac{K}{N}\sum_{j=1}^{N}\sin\left(\left(y[j] + \frac{k1[j]}{2}\right) - \left(y[i] + \frac{k1[i]}{2}\right)\right)\right) \qquad i = 1 \cdots N$$

$$k3[i] = h\left(\omega[i] + \frac{K}{N}\sum_{j=1}^{N}\sin\left(\left(y[j] + \frac{k2[j]}{2}\right) - \left(y[i] + \frac{k2[i]}{2}\right)\right)\right) \qquad i = 1 \cdots N$$

$$k4[i] = h\left(\omega[i] + \frac{K}{N}\sum_{j=1}^{N}\sin((y[j] + k3[j]) - (y[i] + k3[i]))\right) \qquad i = 1 \cdots N$$

$$y_{new}[i] = y[i] + \frac{k1[i]}{6} + \frac{k2[i]}{3} + \frac{k3[i]}{3} + \frac{k4[i]}{6} \qquad i = 1 \cdots N$$

The calculated $y_{new}[i]$ is the equivalent to the phase for oscillator i (ref. the Kuramoto model above), advanced one step size h. The Runge-Kutta method for calculating y_{n+1}, which is y_n incremented with a stepsize h ahead in time, is illustrated in the picture below.

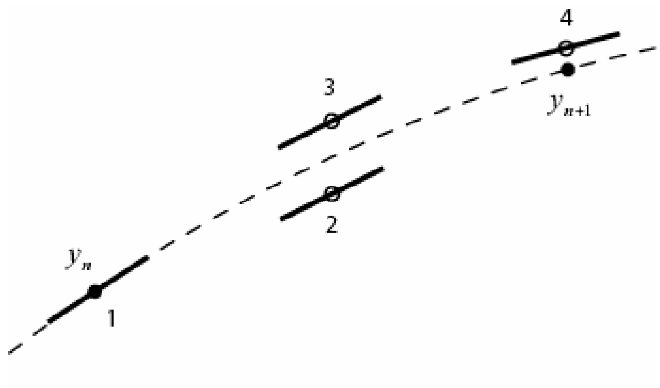


Figure 3: Fourth-order Runge-Kutta method. In each step the derivative is evaluated four times: once at the initial point, twice at the midpoints, and once at a trial endpoint. From these derivatives the final function value is calculated, shown as a filled dot.

The figure above illustrates the Runge-Kutta method of fourth order. For each step, the derivative is calculated four times: once at the initial point, twice at the midpoints and once at a trial endpoint. From these derivatives the final function value (shown as a filled dot) is calculated[31].

Implementation in a one-dimensional space

The Kuramoto model was implemented in a one-dimensional space in order to test the behavior of coupled nonlinear oscillators. This implementation was using an array-based structure for the oscillators.

The verification of this code was done by code reviews and printouts to the screen. The oscillators tended to sync when the coupling strength was increased, and in the same way they tended to fall out of sync when the coupling strength was decreased.

In these first tests of the behaviour of the Kuramoto model, all oscillators were all equally coupled to all the others. In such a system all oscillators "listens" equally on all of the other oscillators, independently of distance, etc. Having the oscillators lined up on a "line", as in an array, also made the logic structure of the oscillators a bit complex when a having the mapping to the physical space in mind. Therefore, a redesign of the structure for the oscillators was done.

Structure for oscillators

Initially, all oscillators were implemented in an array, as described in the previous subchapter.

$$oscillator_0 \quad oscillator_1 \quad \cdots \quad oscillator_{N-1}$$

Figure 4: Oscillators' structure in implementation of Kuramoto model in a one-dimensional space.

In order to logically structure the oscillators in a way that makes representation of the physical space easier and more logical, a two-dimensional structure was introduced. The structure for the oscillators was reconfigured from the form of an array into a two-dimensional grid instead – a matrix. By having the oscillators arranged in a two-dimensional space it got easier to associate each oscillator to a specific location in the room. The new structure also made it less complex and complicated to define neighbours to each oscillator.

$$\begin{matrix} oscillator_{00} & oscillator_{01} & \dots & oscillator_{0(xN-1)} \\ oscillator_{10} & oscillator_{11} & \cdots & oscillator_{1(xN-1)} \\ \vdots & \vdots & \ddots & \vdots \\ oscillator_{(yN-1)0} & oscillator_{(yN-1)1} & \cdots & oscillator_{(yN-1)(xN-1)} \end{matrix}$$

Figure 5: Oscillators' structure in implementation of Kuramoto model in a two-dimensional space.

Implementation in a two-dimensional space

The logical structure of the oscillators was redesigned according to the previous chapter. The oscillators were mapped to a two-dimensional grid to represent their location in the physical space.

This gives the algorithm the following expression:

$$k1\left[i_y\right]\left[i_x\right] = h\left(\omega\left[i_y\right]\left[i_x\right] + \frac{K}{N}\sum_{j=1}^{N}\sin\left(y\left[j_y\right]\left[j_x\right] - y\left[i_y\right]\left[i_x\right]\right)\right)$$

$$k2\left[i_y\right]\left[i_x\right] = h\left(\omega\left[i_y\right]\left[i_x\right] + \frac{K}{N}\sum_{j=1}^{N}\sin\left(\left(y\left[j_y\right]\left[j_x\right] + \frac{k1\left[j_y\right]\left[j_x\right]}{2}\right) - \left(y\left[i_y\right]\left[i_x\right] + \frac{k1\left[i_y\right]\left[i_x\right]}{2}\right)\right)\right)$$

$$k3\left[i_y\right]\left[i_x\right] = h\left(\omega\left[i_y\right]\left[i_x\right] + \frac{K}{N}\sum_{j=1}^{N}\sin\left(\left(y\left[j_y\right]\left[j_x\right] + \frac{k2\left[j_y\right]\left[j_x\right]}{2}\right) - \left(y\left[i_y\right]\left[i_x\right] + \frac{k2\left[i_y\right]\left[i_x\right]}{2}\right)\right)\right)$$

$$k4\left[i_y\right]\left[i_x\right] = h\left(\omega\left[i_y\right]\left[i_x\right] + \frac{K}{N}\sum_{j=1}^{N}\sin\left(\left(y\left[j_y\right]\left[j_x\right] + k3\left[j_y\right]\left[j_x\right]\right) - \left(y\left[i_y\right]\left[i_x\right] + k3\left[i_y\right]\left[i_x\right]\right)\right)\right)$$

$$y_{new}\left[i_y\right]\left[i_x\right] = y\left[i_y\right]\left[i_x\right] + \frac{k1\left[i_y\right]\left[i_x\right]}{6} + \frac{k2\left[i_y\right]\left[i_x\right]}{3} + \frac{k3\left[i_y\right]\left[i_x\right]}{3} + \frac{k4\left[i_y\right]\left[i_x\right]}{6} \; ,$$

In the equations above the indexes i_y and i_x ranges from $i_y = 0\cdots\left(N_y - 1\right)$ and $i_x = 0\cdots\left(N_x - 1\right)$, where N_y and N_x are the size of the matrix in y- and x-dimensions.

Redefinitions of coupling conditions

In the implementations discussed so far, and the tests made with these, all oscillators have been equally coupled to all other oscillators, independently of distance to the others, etc. This soon gives a state where all oscillators fall into sync. To make it likely for the oscillators to sync locally, and to possibly get groups of locally synchronized oscillators, the coupling conditions were redefined. The oscillators' coupling conditions then had to be dependent on the distance to the other oscillators.

Instead of having each oscillator coupled to all the others, each oscillator were redefined to be coupled to only its closest neighbours.

That way, local synchronizations between oscillators are more likely to occur. It would be logic that oscillators that have locally fallen into sync could spread it's synchronization over time to neighbouring cells in the system, which in turn will have an effect on it's neighbours, and so on. Consequently, by having the oscillators being just locally coupled, it is more likely to get local synchronizations spreading through the system, and also to get many different nodes where local synchronizations evolve.

Since the structure of the oscillators has been redesigned according to the previous chapters, into a two-dimensional space, it is easier to define the neighbours for each oscillator compared to having a one-dimensional structure as in the first tests. The two-dimensional structure makes it straightforward to define each oscillator's neighbours as in the picture below.

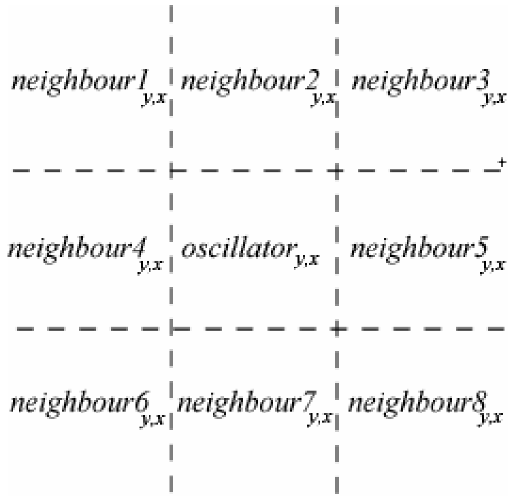


Figure 6: Definition of neighbors in a two-dimensional structure for the oscillators.

The neighbours in the picture above also have different grade of affect on the oscillator, depending on their distance to the oscillator. Neighbour number 1, 3, 6 and 8 are more distant than number 2, 4, 5 and 7. The neighbour number 1, 3, 6 and 8 have a distance to the oscillator a factor $\sqrt{2}$ times the distance of the neighbours 2, 4, 5 and 7. Thus, the influence from neighbour 1, 3, 6 and 8 becomes $1/\sqrt{2}$ times the influence from the neighbours 2, 4, 5 and 7. This relation is illustrated in the picture below.

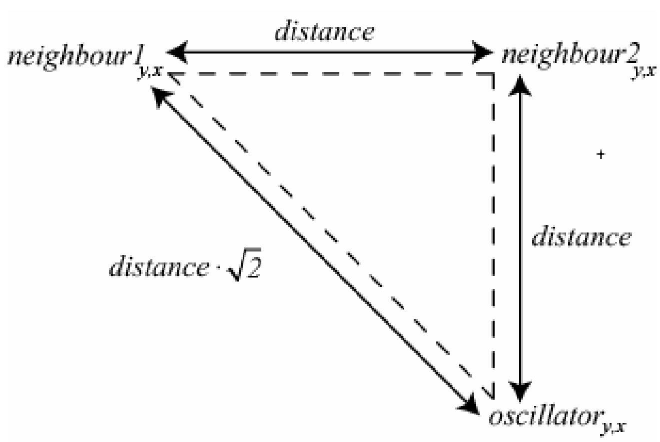


Figure 7: The neighbour 1, 3, 6 and 8 are more distant than neighbour 2, 4, 5 and 7, ref. the Pythagoras' theorem.

Tests have shown though, that the inclusion of coupling to neighbours number 1, 3, 6 and 8 is less important for the result. In fact, one couldn't tell any difference from having these four neighbours being coupled to each oscillator or not. For that reason, the coupling conditions were redefined again, into having only the neighbour number 2, 4, 5 and 7 being coupled to each oscillator, i.e. only the very closest neighbours are coupled to each oscillator.

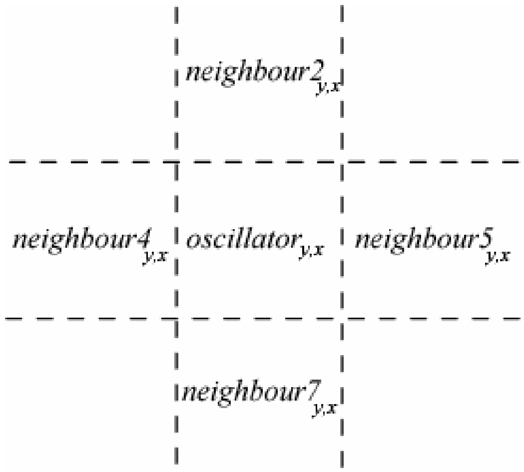


Figure 8: The closest neighbors to each oscillator.

The implementation of the Kuramoto model, with each oscillator coupled to only its closest neighbours, includes a function for handling boundary conditions for the matrix.

Boundary conditions

To make the system independent of the matrixes' boundaries, a function handling periodic boundary conditions was introduced. This gives a more general implementation, making indexing in the matrix independent on the position in the matrix. Indexing outside the matrix boundaries makes the indexing being folded back into the matrix. The implementation also becomes more flexible for extensions of the coupling conditions, if an extended coupling to further neighbours as well would be interesting in the future.

The implementation of the Kuramoto model is at this stage modified to only calculate over the necessary cells in the matrix, i.e. the closest neighbours. Not only does this give the system a more interesting behaviour as local synchronizations in the grid are more likely to occur and where groups of locally synchronized oscillators possibly can spread their sync patterns through the grid, but also is this implementation more effective, i.e. faster, as calculations are made only over the necessary cells in the matrix. In other words, the calculations of the coupling impact for each oscillator are only made for the closest neighbours and not for every cell in the matrix, which consequently results in a faster program.

Verification of the Kuramoto algorithm

The first test was verified with code reviews and text printouts to the screen. But to better see what happens when the oscillators are coupled to only their closest neighbour, a graphical representation of the system would better illustrates the behaviour of the system. As mentioned in the previous chapter, it is likely that groups of neighbouring oscillators fall into sync. To best illustrate this, graphical representations of the system are introduced.

Verification of coupling impact on oscillators

To verify the behaviour of the system in a graphical way, GnuPlot was first evaluated. A test environment for GnuPlot was set up. The following picture shows a GnuPlot print, including the phases for the different oscillators in the matrix.

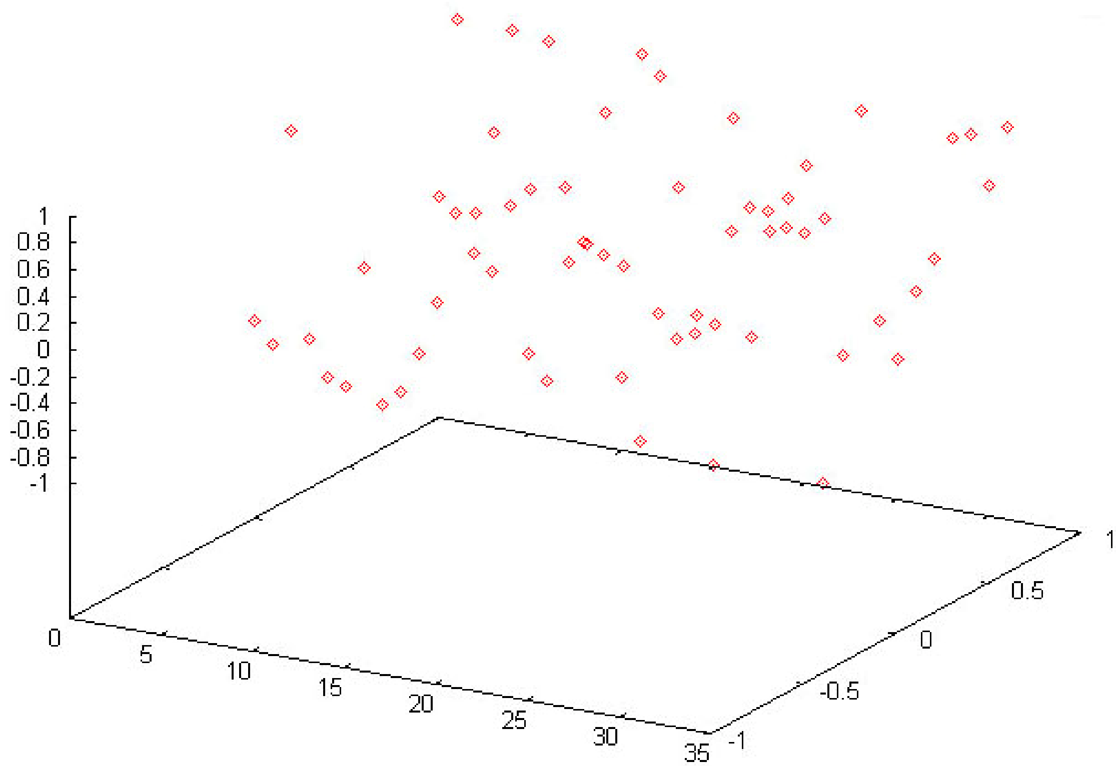


Figure 9: A GnuPlot print of the oscillators' phases.

The graphical representation achieved with GnuPlot was shown to be somewhat inadequate for the visualisation of the behaviour of the system. To get a better visualization of the oscillators' states, alternative plotting options were evaluated, and the program was subsequently redesigned to support OpenGL graphics.

OpenGL plotting

The idea with an OpenGL test printout window is to let squares represent each element in the matrix with a colour indicating the corresponding oscillators' state, i.e. the phase for each oscillator. That way it gets obvious if some adjacent oscillators fall into sync.

To examine this in an appropriate way, the number of oscillators should be large enough for being able to visually see how synchronization patterns in the grid evolves over time. The number of oscillators was thus increased to a number of 1024, which gives a 32 by 32 grid.

In order to visualize how the phase for each oscillator changes over time, the phase is printed out as a grayscale square, ranging from black (representing $\sin\left(y\left[i_y\right]\left[i_x\right]\right) = -1$) to white (representing $\sin\left(y\left[i_y\right]\left[i_x\right]\right) = 1$).

First of all, the phase for the individual oscillators, $\sin(y[i_y][i_x])$, is plotted over time, in order to check that the plotted values follow the expression of a typical sinus wave. This test is shown in the picture below.

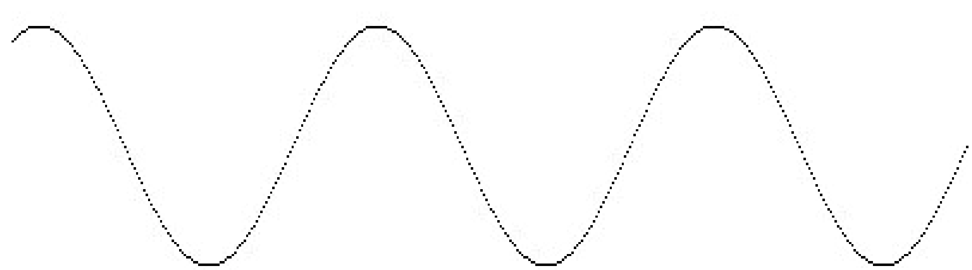


Figure 10: An OpenGL plot of the phase for one of the oscillators, plotted over time

The picture above indicates that the phase, $\sin(y[i_y][i_x])$, for each single oscillator follows the expected expression. The plot for one single oscillator also gets the expected expression when it tries to synchronize to the other oscillators' frequencies. The curve does then no longer look like a pure sinus wave, as the oscillator's phase tries to synchronize to its neighbour's phases. The plot below shows the phase for one oscillator when it is about to get synchronized to its neighbours. The single studied oscillator has a different natural frequency than its neighbours (10 instead of 1).

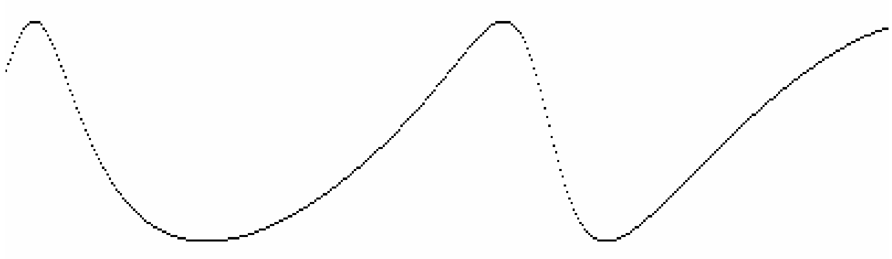


Figure 11: The phase for one single oscillator, in a state where it tries to synchronize to surrounding oscillators' phases.

Now, the behaviour of all oscillators will be plotted to verify the implementation of the Kuramoto algorithm and to examine how the system behaves in various different situations.

The implementation of the OpenGL plotting functionality is also given support for interaction with the system during runtime. Coupling strength and step length can be changed when running the program, by keyboard interface.

In the following figures, a test is made with 1024 oscillators (32x32), where the coupling strength to the oscillators' neighbours is increased from a near-to-zero value up to a value where all oscillators eventually tend to all be in synch.

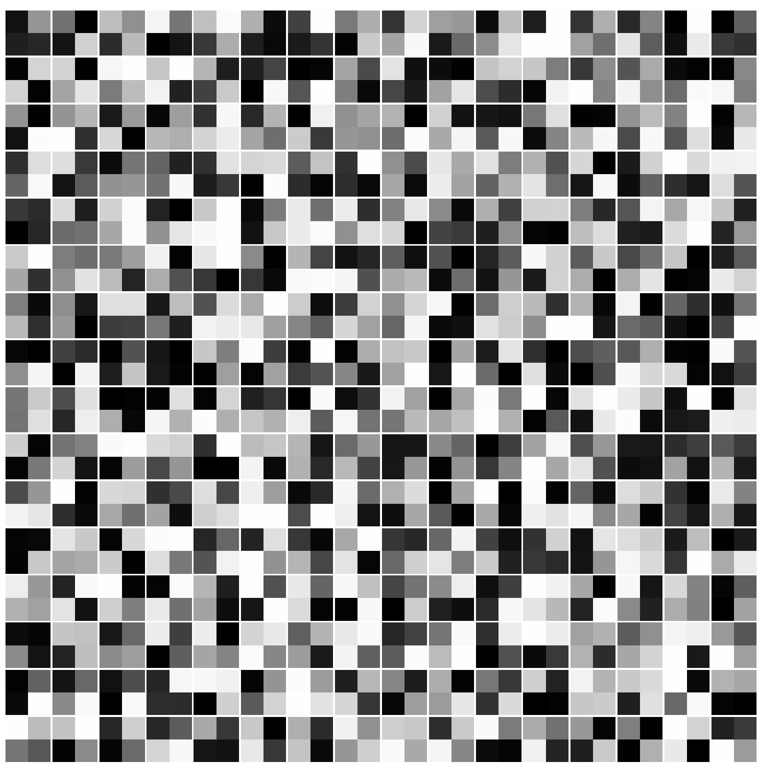


Figure 12: The coupling strength is here 0. The state for the different oscillators here varies independently of the others. No correlation between the oscillators' phases is visibly detectable.

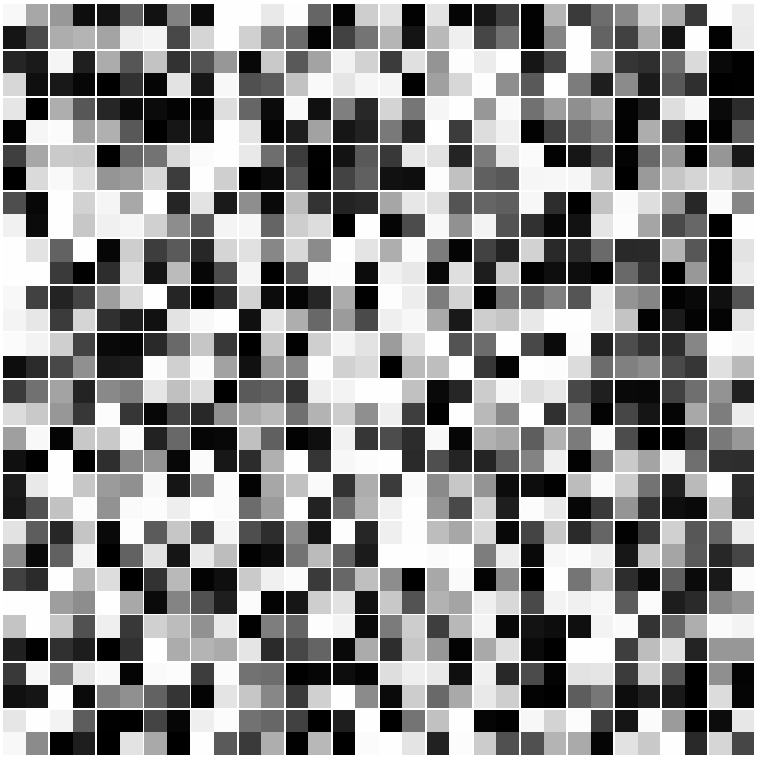


Figure 13: Here the coupling strength has been increased to 52.43. Some local synchronizations start to evolve, but to a very limited degree. These local synchronizations seem to spread vaguely in the grid.

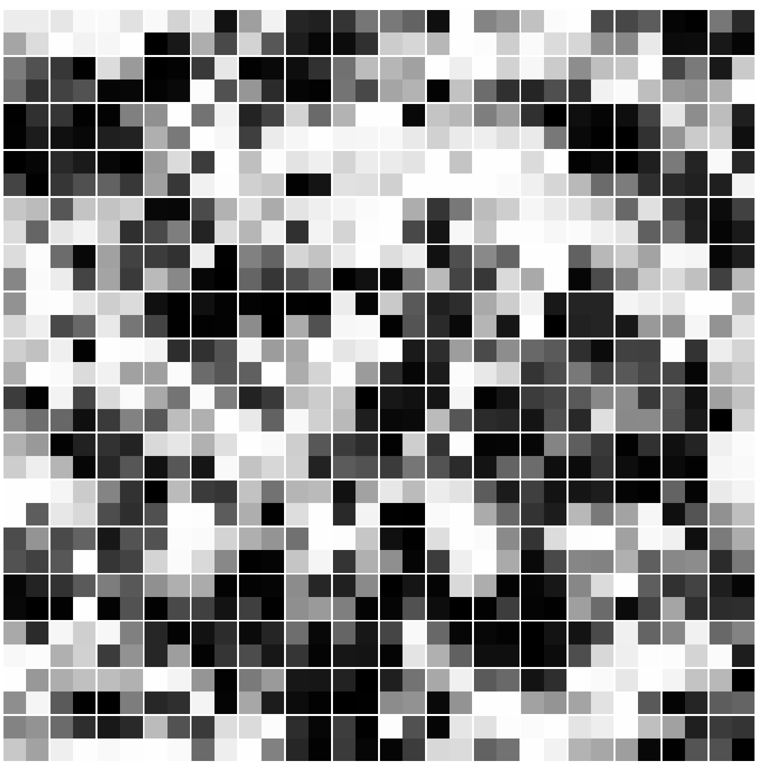


Figure 14: The coupling strength is here increased to 419. Now clear local synchronizations evolve. Neighboring cells create "clouds" that moves through the grid.

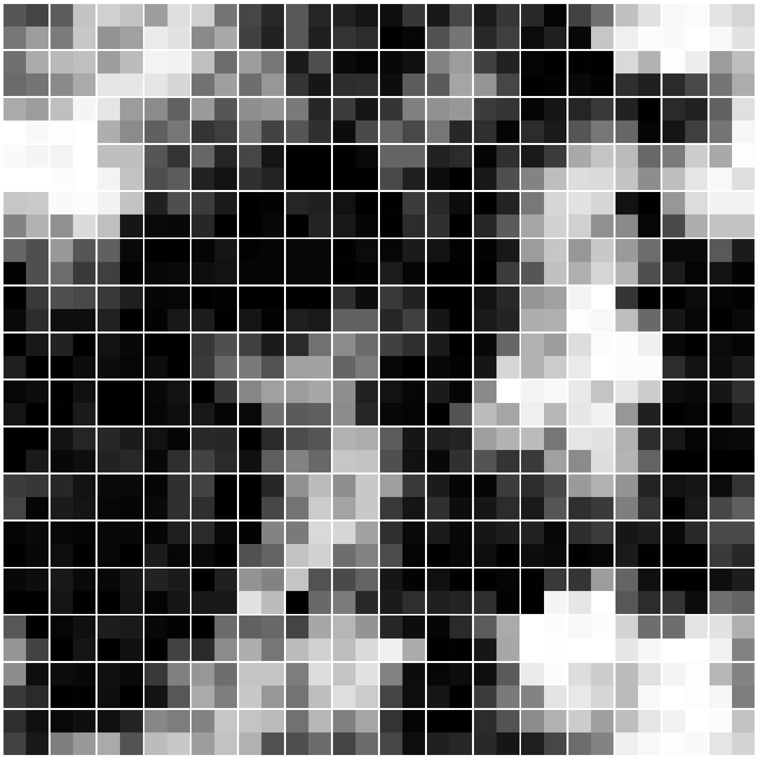


Figure 15: Coupling strength is here increased to 839. The "clouds" are now even clearer and the difference between neighboring cells is smaller.

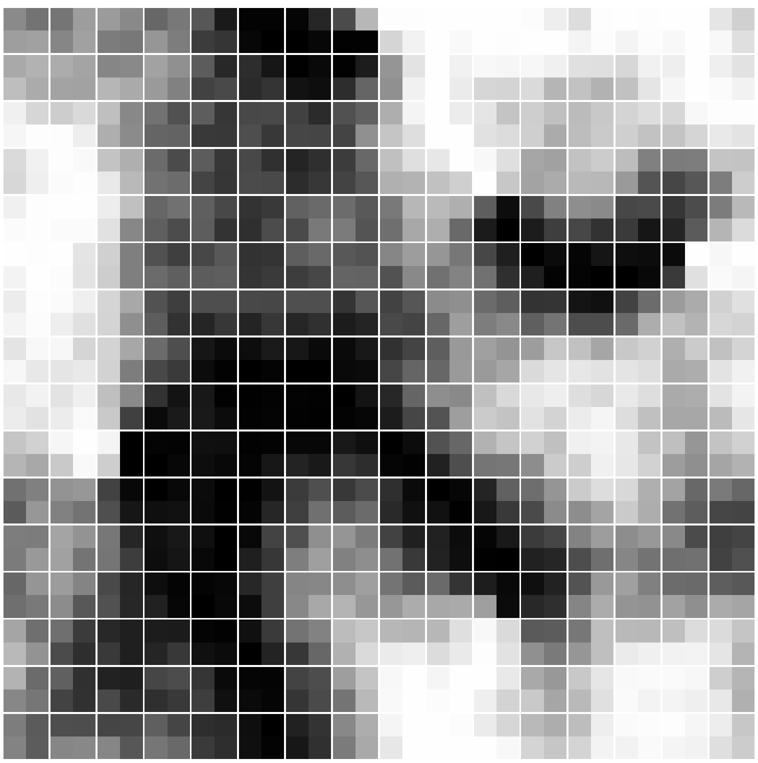


Figure 16: Here the coupling strength is increased to 1678. The synchronizations are even clearer and the differences between the phases for neighboring oscillators are lesser as well.

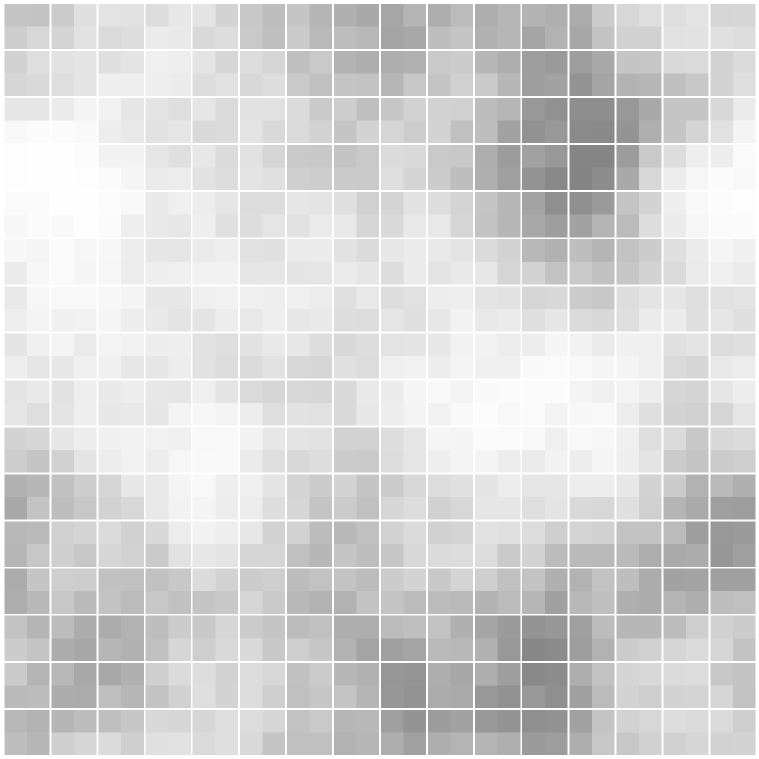


Figure 17: The coupling strength is here increased to a value of 3355. All oscillators are now coupled to the others to some degree. Still there are significant "clouds" indicating that local groups of oscillators are more tightly synchronized than the oscillators in the rest of the grid.

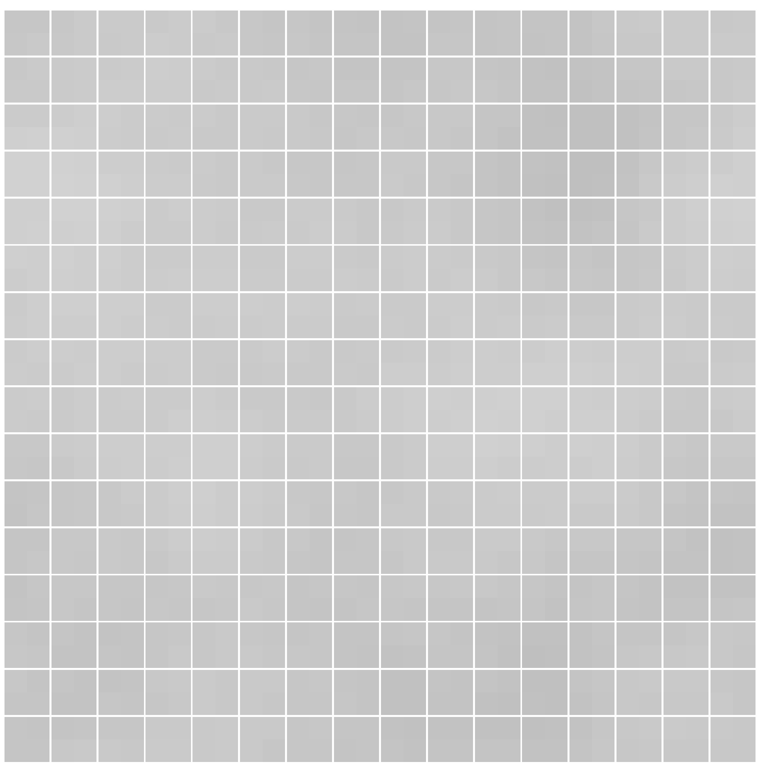


Figure 18: Here the coupling strength is even more increased, to a value of 26844. The oscillators look to be equally coupled. The oscillators' phases all vary over time following all others' phases.

Having the step length decreased to a fraction of the value used in the pictures above, slows down the changes in the pictures, since the new states are then calculated for a smaller increment in time. This makes changes in the picture easier to determine as a slower motion in the picture is achieved. The pictures below are printouts of changes over times when the step size is decreased to a very small value (0.000078, instead of 0.01 as in the pictures above).

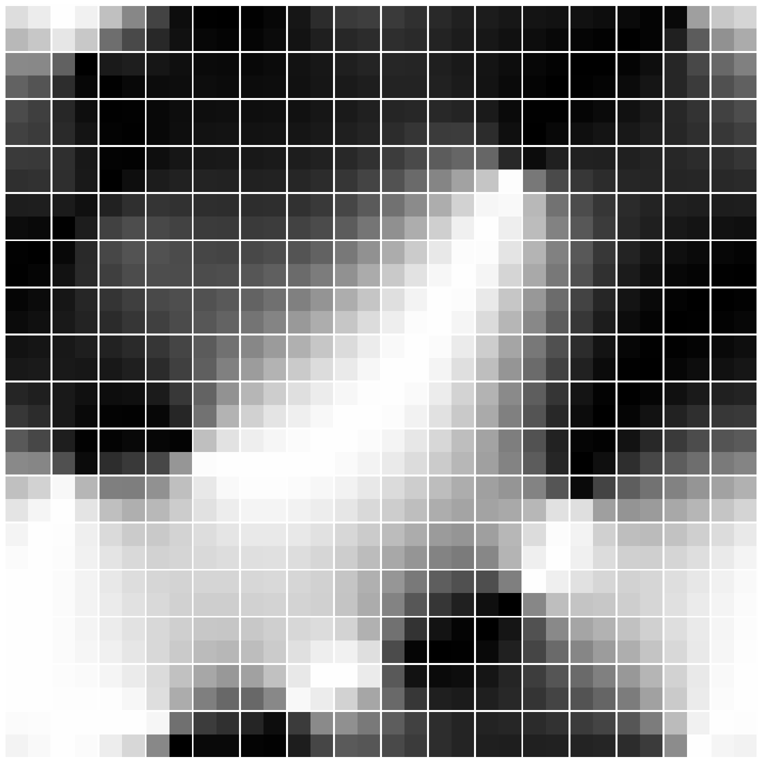


Figure 19: Having the step size decreased to a very small value makes changes in the picture evolve very slowly, which makes determining variations of oscillators' phases easier to determine.

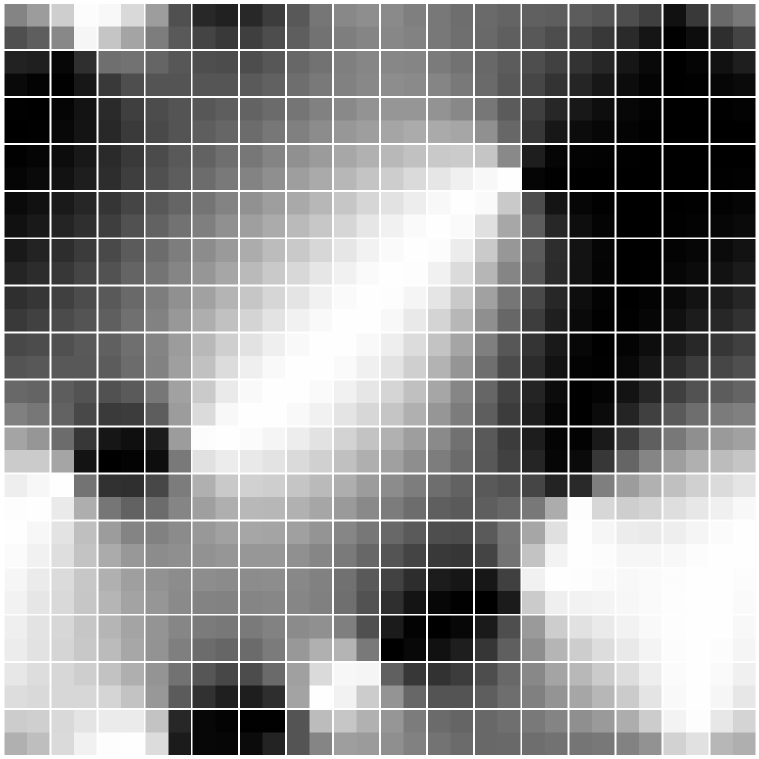


Figure 20: The variations of the oscillators' phases are here incremented, but the variations from the previous picture are slowed down by decreasing the step length for the Runge-Kutta algorithm. This is one way of making local synchronizations easier to visually determine.

As illustrated above the tempo in the variations for the oscillators' states can be varied by increasing or decreasing the step length. Though, by increasing the step length to a too large value will result in inaccurate approximations. The picture below illustrates the phases for the oscillators when the step length is too big.

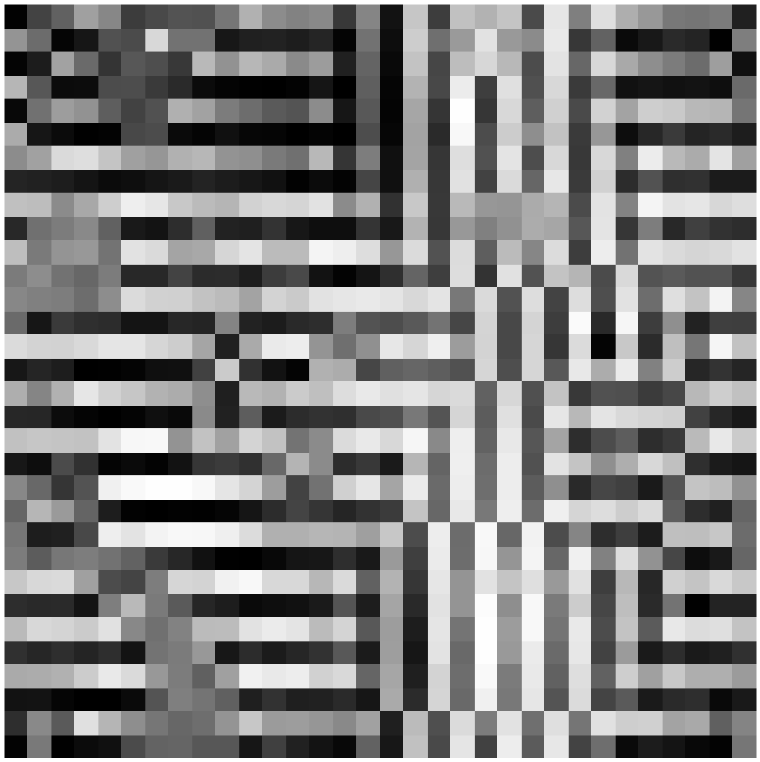


Figure 21: The step length has been increased to a too large value and the Runge-Kutta algorithm does no longer give an accurate result.

The Runge-Kutta algorithm will give the same inaccurate result when the coupling strength is increased to a too large value. The value of the coupling strength and the step length are thus intimately connected to each other. Tests have shown that the sum of the coupling strength and the step length needs to be under a certain level in order for the Runge-Kutta model to behave accurately. This phenomenon is affected by the step length and coupling strength parameters, and not by the start values for the oscillators or their natural frequencies.

The limits for the coupling strength and the step length, in order to achieve accurate behaviour for the Runge-Kutta algorithm, are shown in the table below.

Coupling strength	***Step length***	$K \bullet h$
26844	**0.01**	**268,44**

Table 1: The sum of the coupling strength and the step length need to be under 268 in order to receive accurate results from the Runge-Kutta model, referring to these tests done with the 32x32 matrix.

However, further tests shows that the relations described in the table above are also dependent on the size of the matrix. Tests were done with matrixes of size 2x2, 4x4 and 16x16 as well. The following results were received when using various sizes of the matrix.

Coupling strength	***Step length***	***Matrix size***	$K \bullet h$ **limits**
26844	0,01	32x32	**268,44**
6710,89	0,01	16x16	**67,1089**
838,86	0,005	4x4	**4,19**
104,86	**0,01**	**2x2**	**1,0486**

Table 2: Limits for receiving accurate results from the Runge-Kutta algorithm.

In the figure above it is shown that the limit for K and h depends on the size of the matrix. Though, if the limits for $K \bullet h$ is divided by their corresponding number of elements in the matrix, they all give a constant value that can be used as a limit independently of the size of the matrix. The following table shows the constant limit received when the $K \bullet h$ limits are divided by their matching matrix sizes.

Coupling strength	***Step length***	***Matrix size***	$K \bullet h$ **limits**	$\frac{K \bullet h}{N}$
26844	0,01	32x32 $\rightarrow N = 1024$	268,44	**0,262148**
6710,89	0,01	16x16 $\rightarrow N = 256$	67,1089	**0,262144**
838,86	0,005	4x4 $\rightarrow N = 16$	4,19	**0,261875**
104,86	**0,01**	**2x2** $\rightarrow N =$ **4**	**1,0486**	**0,26215**

Table 3: The limits for coupling strength and step length are dependent on the size of the matrix that is used. A constant limit for getting accurate results from the Runge-Kutta algorithm is received when taking the matrix size into account as well.

Consequently, the expression $\frac{K \bullet h}{N}$ will be used in the program as a parameter that is to be held under a value of $0,26$ in order to get accurate results from the Runge-Kutta algorithm.

The studies and tests made of the Kuramoto algorithm using a large size of the matrix have given better understanding of the algorithms behaviour than if a smaller size of the matrix had been used in these tests. Based on this knowledge of the algorithm's behaviour, the size of the matrix is now scaled down to a size more suitable for a set of sounds. If a too large number of sounds would be used the resulting output would get too blurred for the wanted result.

Mapping the Kuramoto model to sound

The idea is to let each oscillator's state control the sound. Each oscillator has a sound associated to it. By keeping track on the current state (i.e. phase) for each individual oscillator, the corresponding sound is to be played when the oscillator's state has reached a certain level. By defining a threshold level for the phase, the oscillator's corresponding sound is triggered when the threshold is passed.

Each sound is triggered only when the threshold is passed and entered from a "lesser" state. This is illustrated in the figure below.

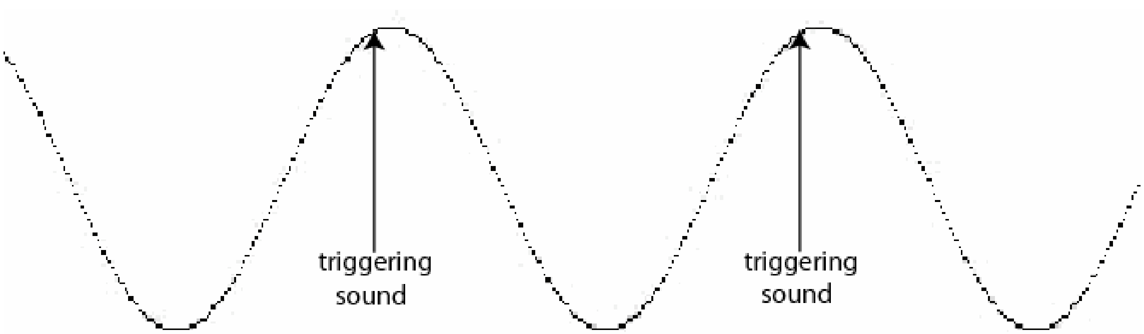


Figure 22: Sounds are triggered when a certain threshold level is reached. In this figure, the threshold level is set to 0,99.

When a sound is triggered the whole sound is played. If, which probably won't ever happen, a new triggering event is reached before the previous sound has finished playing, the new triggering event will be ignored. When the oscillator's state has fallen below triggering level again *and* when the sound has finished playing, the sound is ready to be triggered again.

Sound

Sound programming

Implementation environment

The implementation is done in C++ in the Microsoft Visual Studio .NET environment and run in win32 console application mode.

Interfaces and libraries

The *PortAudio*[32] API is used for playing sounds. *PortAudio* is an audio I/O library that provides an interface for real-time audio streaming services in the form of a C-language API.

Libsndfile[33] is used for reading sound files. Libsndfile is a C library for reading and writing files containing sampled sound (such as WAV-files).

Sound files

All sound files used in this project have Microsoft WAV format (little endian), 16 bit mono files with 44100 kHz sample rate.

Program structure

The implementation structure builds on two threads running continuously in parallel to each other, as two loops. The first loop, *main loop*, handles all updating of the oscillators' statuses over time, i.e. increments the Kuramoto algorithm and updates the statuses for all oscillators. The main loop also handles graphical/textual output to the display. The second loop is a callback function that handles all the sound processing. This loop takes care of sending accurate sound to the output buffer whenever sounds are to be played.

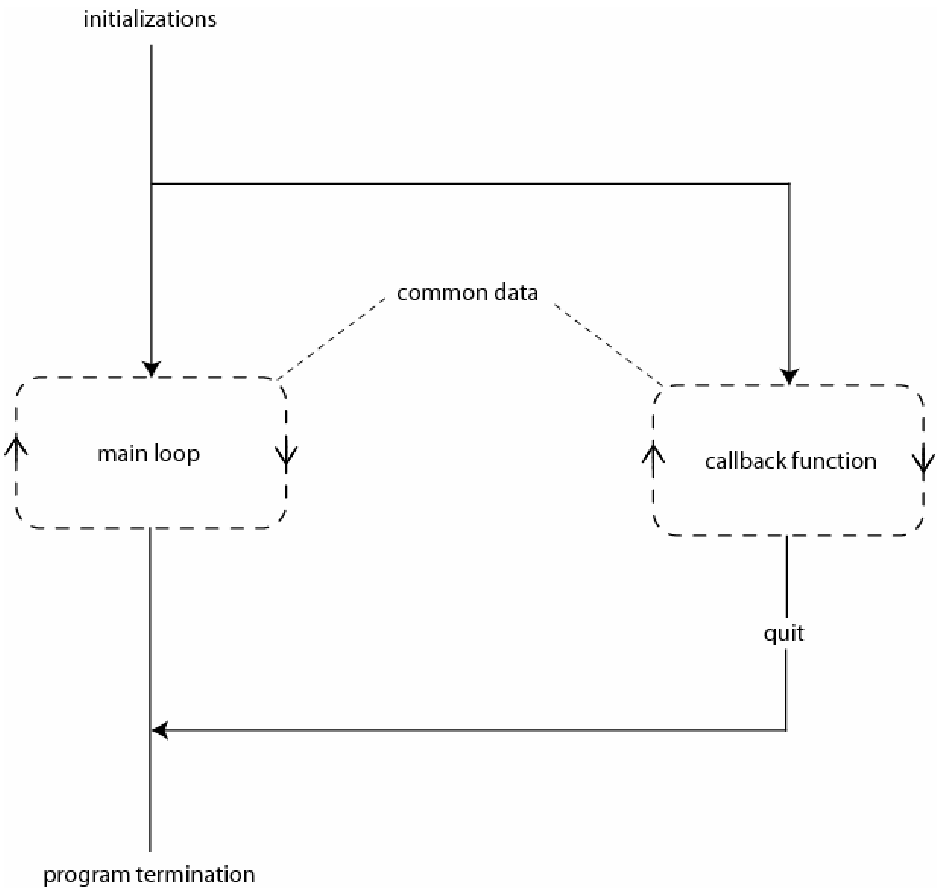


Figure 23: The program runs as two threads. A main loop handles all status updating and graphical output, and a callback function handles audio output.

The two loops are started as soon as all initializations are done, and run until the program execution is terminated by a user command (Esc key).

Both loops have constant access to data such as status information and oscillator data. The main loop updates the status data, which is read by the callback function.

Initializations

All start values for the Kuramoto algorithm are initially set, as well as the oscillators' natural frequencies.

The step length and the coupling strength are also given start values. These values are not constant, i.e. they are continuously able to be changed during runtime.

All status variables are initially reset.

Any window outputs are initialized.

All oscillators' corresponding sounds are read from files and stored into buffers.

The picture below shows the structure for the audio buffers, which is filled with data in the initialization process and later used by the callback function to read the oscillators' corresponding sounds.

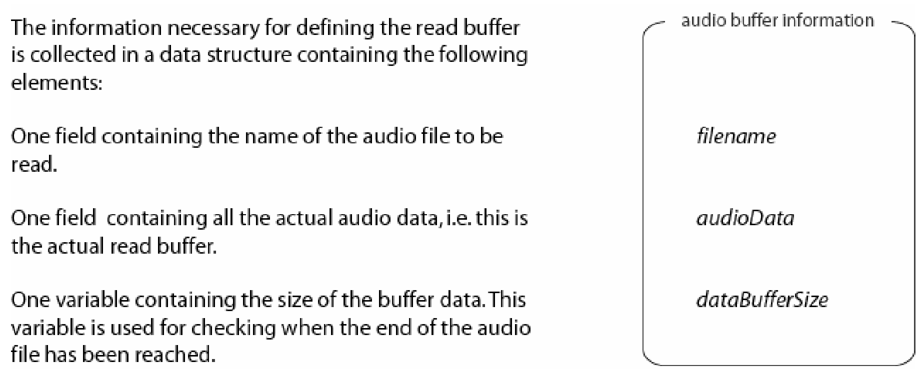


Figure 24: The structure for the audio buffers.

After all sounds are read, PortAudio initializations are done and a stream is opened and started in order to make the callback function ready to process audio data.

The main loop is then started.

Main loop

The main loop continuously increments the Kuramoto algorithm and thereafter updates all of the oscillators' status variables. The callback function is given a part of the processor time, in order to make sure that the callback function gets enough time to process all possible data.

The status data that the main loop is continuously updating is listed in the figure below.

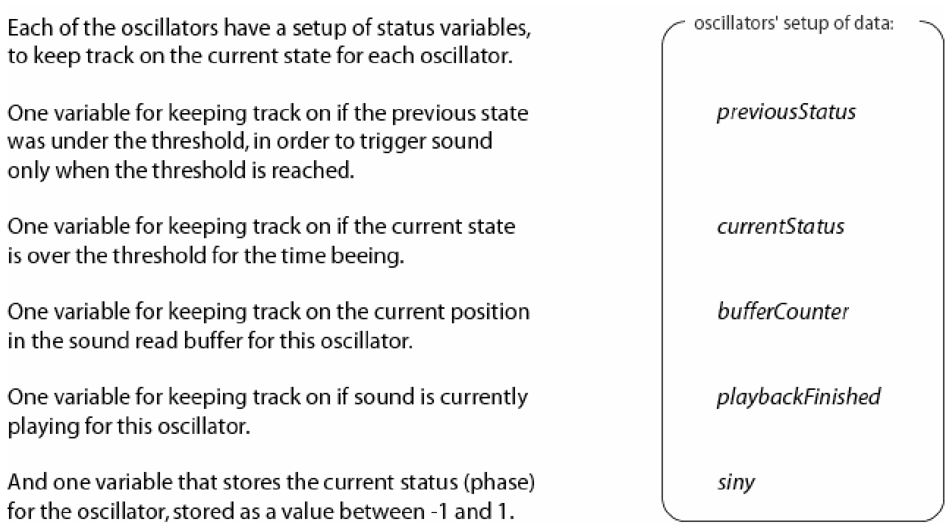


Figure 25: The status data associated to each oscillator.

As soon as an oscillator's status has reached the threshold level, the main loop sets the status variables to flag that the oscillator's corresponding sound is to be output. This output is then instantly handled by the callback function.

Callback function

The callback function continuously calls a function, *calculateOutput*, in order to get the next sample that is to be written to the output buffers. This is illustrated in the figure below.

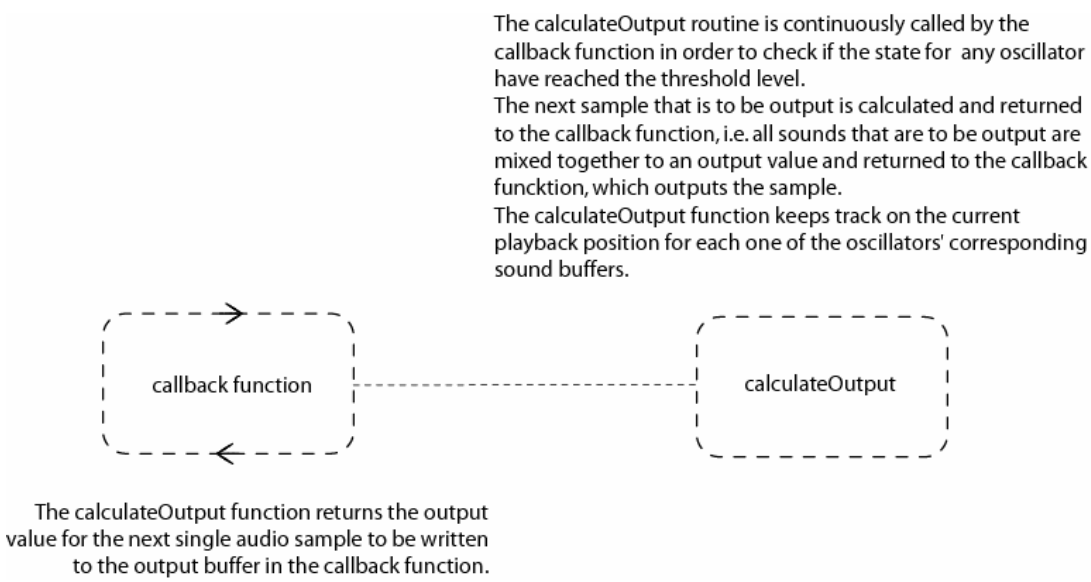


Figure 26: Processing of audio data is handled by the callback function, which continuously calls a function to mix the next audio sample to be output.

As long as no sounds shall be played, the calculateOutput function returns zeros to the callback function. As soon as one oscillator's state have reached the threshold level, its sound is read from the corresponding sound buffer and the calculateOutput function keeps track on where in the sound buffer to read next, and returns the current value to the callback function which outputs the audio.

If more than one sound is to be output, the calculateOutput function mixes the sounds together and returns the mixed value to the callback function. The corresponding sound buffers' read pointers are used to select the samples that is to be mixed together. The calculateOutput function returns the mixed output value and updates the pointers to the buffers read from. The mixing process is illustrated in the figure below.

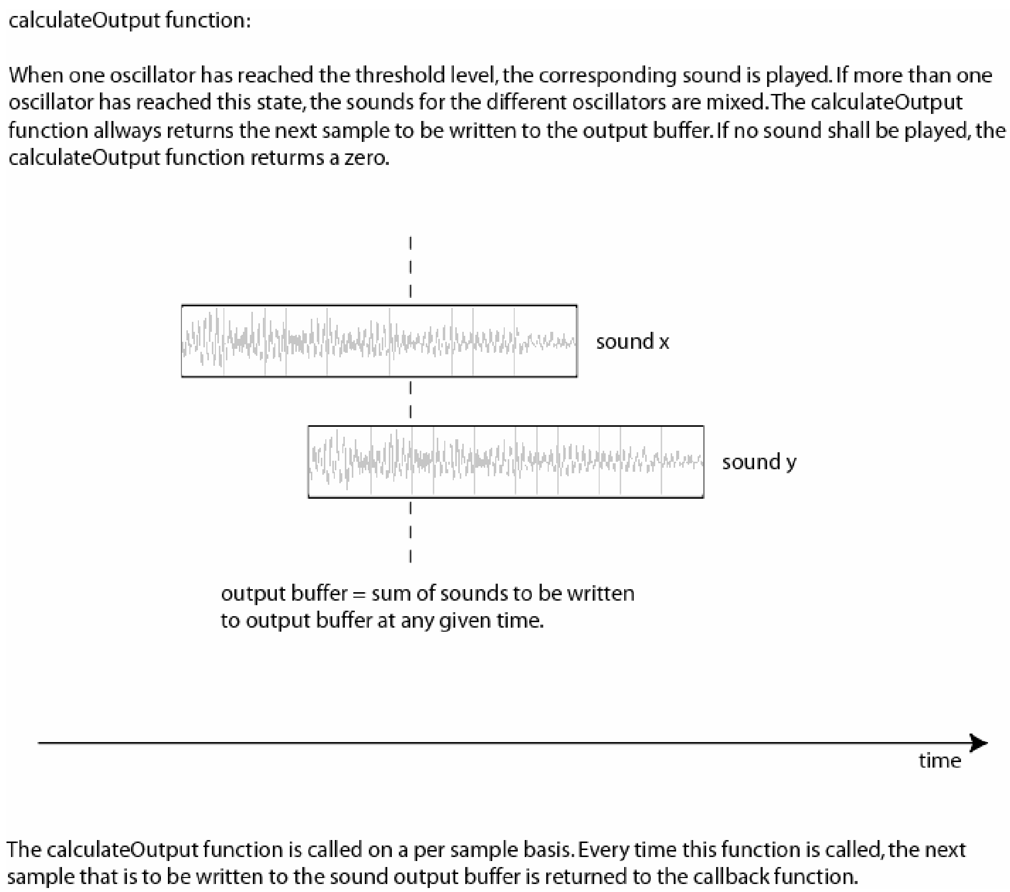


Figure 27: Mixing of several sounds.

As soon as one whole sound has been played back, the calculateOutput function switches off the corresponding flag that indicates that the sound shall be played back. This flag is then lit again (in the main loop) as soon as the oscillators' state has reached the threshold level again.

Installation

Design of installation

The installation is made up of the results achieved from the various experiments and elaborations done with the Kuramoto model, as explained in the previous chapters. The number of sounds that are used, the control of the algorithm and the sounds themselves to be used are parameters that have been chosen along the way based on numerous tests that have been carried out.

Number of oscillators

For the number of oscillators to be used, various setups have been evaluated. To find a suitable number to use in the installation, tests have been done with as few as two oscillators which is the minimum number of oscillators to use according to the concept of entrainment, up to a number of 1024 when the output of sound got too blurred for the wanted expression. Out of all different sizes for the number oscillators to use, the numbers between 16 and 256 appeared to be the most interesting.

The number of oscillators to use has been evaluated with different setup of sounds as well – the tests have included customizing of the used sounds as well, i.e. when a larger number of oscillators are used it has shown that shorter sounds are more interesting to use.

Fewer than 16 oscillators have shown to be not that interesting; both in terms of the effect of the algorithm which is then not that obvious, and the number of sounds themselves which gets a little too few for giving a musically interesting result.

Conversely, having a number of oscillators larger than 256 gives a less interesting result as well, as the output becomes too blurred for the wanted expression.

When using a size of 256 oscillators, very short sounds were best suited to use. Having this many sounds and having them all very short gave the most appealing results of all tested combinations. This was the case where the phenomenon of entrainment got most palpable. Hence the number of 256 oscillators was appointed to be used in the installation as it made the entrainment component appear most clearly in this case.

Sounds

Similarly to when choosing the number of sounds to be used, the sounds themselves have also been selected by designing and testing different sounds and different combinations.

The sounds to use have been designed with the number of oscillators to be used in mind. Shorter sounds have shown to be appropriate to use when larger amount of oscillators are used. Various sets of sounds have thus been created, each with the context in which to use them in mind. For a set of 16 or 36 oscillators, the typical set of sounds that have been created have been similar to percussion sounds, though many other have also been tried out but with less interesting result. For a larger set of oscillators, shorter sounds have been created.

Many different *kinds* of sounds have been evaluated; natural sounds from frogs and crickets and other insects, human voices, claps, acoustical instruments, percussion sounds and many different electronic sounds.

Finally, the sounds that have been selected to be used for the installation are a set of designed very short electronic sounds. When using this large number of oscillators, the short sounds have shown to give the most sonically interesting result. Then the expression of the installation becomes a cacophony of sound grains that clearly, audibly as well as visually, demonstrates the nature of coupled oscillators.

Control of algorithm

Various possible ways of controlling the algorithm has been evaluated; such as sensors in the room, a graphical user interface, buttons, etc. However, the chosen method for controlling the algorithm is to introduce some controlled randomness for two parameters that are used to control the Kuramoto algorithm; the coupling strength between each oscillator and its neighbours, and the step length in the Runge-Kutta function which is varied in order to control the timescale of the periodicity for the oscillators, i.e. the speed of the rhythmic pattern.

A number of different states has been defined, where the level of coupling strength to surrounding oscillators gives clear differences in the level of entrainment. These states are chosen on the basis that they represent levels in the coupling strength spectra where synchronizations between oscillators emerges; either weakly, where oscillators are about to lock to a common frequency, or where synchronizations locally evolves and eventually can spread throughout the grid.

The algorithm will often need some time before potential synchronizations eventually occurs. Therefore, the algorithm is run in the same state, i.e. same coupling strength, for a couple of minutes before it possibly changes state. Whether the coupling strength is changed or not is determined randomly. Every other minute the coupling strength used in the Kuramoto algorithm *can* change state and does so with the likelihood of 0.5.

The tempo in the rhythm is also altered with controlled randomness, in order to give the installation an ever changing expression. This is done by changing the step length used in the Runge-Kutta function, but always keeping the step length under the limit for receiving accurate results as described in the 'Verification of the Kuramoto algorithm' chapter.

That way the installation continuously changes its expression – from barely listening to its surroundings to a level where all individual oscillators tend to synchronize, and from very slow rhythmic changes to more frenetic beats. This way the system becomes autonomous, and continuously changes its musical expression and rhythmic structures.

There is support for controlling the system in other ways, though; via keyboard and GUI, but these features were only used in the design and test phase of the project and are used in the final installation.

The figure below shows the final design of the installation.

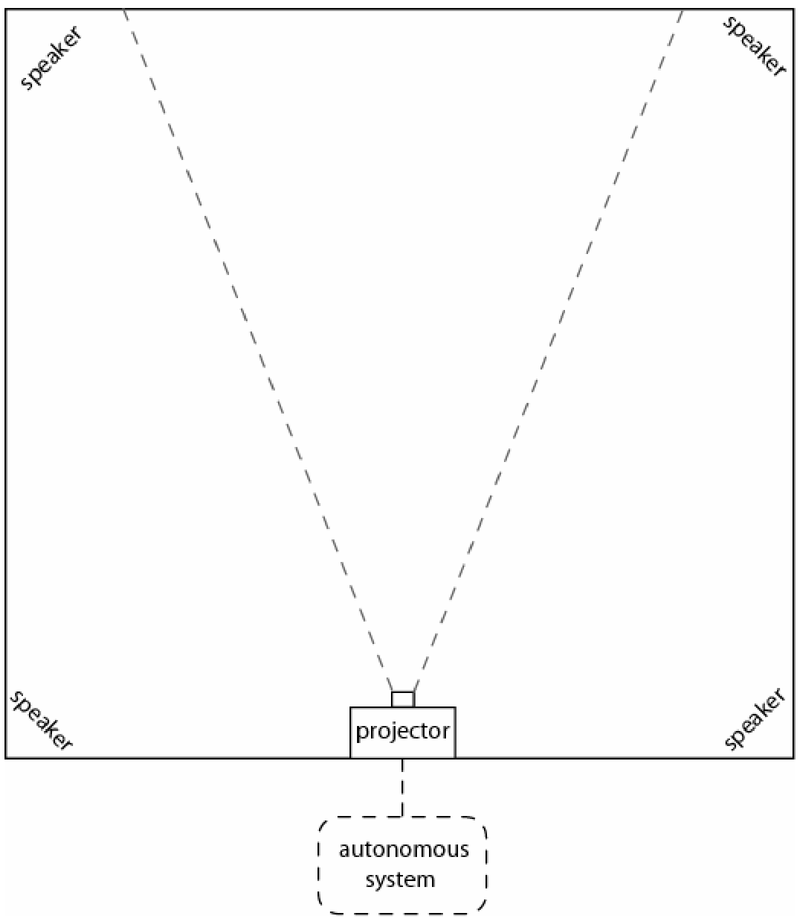
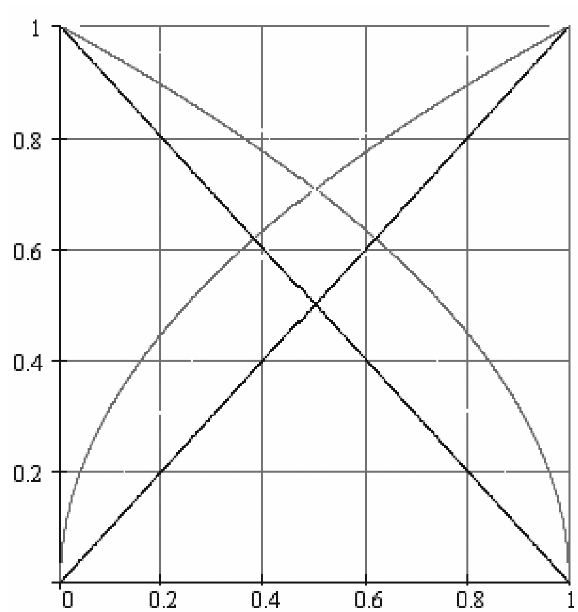


Figure 28: The design of the installation. Four speakers are used to create a surround sound in the room.

The installation is using a projector to display a graphical representation of the system on one of the walls in the room, and one speaker in each corner of the room. The logical structure of each oscillator's position in the room is mapped to its physical position in the room by distributing the sound between the different speakers using a panning function. This way, the visitors to the installation gets surrounded by the sounds of the different oscillators.

Panning of sound

The sound is distributed between the speakers according to the curves below. In order to get a constant sound level the upper curves in the figure below are used.



The sounds for the different oscillators are thus distributed between the left and right speakers according to the following formula; left speaker gets a factor $\sqrt{x}$ of the signal, where x is the oscillator's logical position in the physical space and the right speaker gets a factor $\sqrt{1-x}$ of the oscillator's sound. The sounds positioned between the speakers will thus get the same level as the sounds positioned in the corners. In the same way the sounds are positioned between the front and rear speakers. This way the oscillators' sounds are mapped to their corresponding spot in the room.

End discussion

The initial idea for this project was to do a sound art installation that was based on the way frogs synchronize their singing to each other and how crickets tend to chirp in unison. The more I read and thought of this phenomenon, and at the time I was also reading Malcolm Gladwell's book *The tipping point,* the phenomenon of synchronization between individuals, and different processes or *oscillators*, appeared to be an observable fact in a far wider range of situations than I first imagined. The phenomenon of synchronizations between individuals appeared to in a way also apply to the way people get influenced by their surroundings, which makes people conform and adopt their behaviour to the current situation they are in and the people that surround them, as illustrated in Stanley Milgram's experiment, the Stanford Prison experiment, Solomon Asch's experiment and theory as Malcolm Gladwell's *The tipping point.*

Based on the tendencies observed in people's social behaviour, and its noticeable similarities to the interaction between frogs, as well as many other primitive organisms, this project has got two artistic aims; the first aim has been to let the behaviour of the sounds in this sound art installation be a metaphor for people's tendency to conform, so that the sounds influence each other in a way similar to how people adjust their behaviour to their current surroundings (and how frogs synchronize their singing). The other aim for the project has been to experiment with new ways for musical expression, to elaborate with rhythm in search for a musical expression that I find interesting; by using a model that describes the phenomenon of coupled oscillators as a base for the sound installation.

The final installation got the shape and expression that I aimed for. A set of sounds, each sound representing an individual, are being influenced by each other to a certain degree, creating sounds and rhythms that are strongly characterized by the nature of coupled oscillators explicitly – sounds and rhythms which could not be achieved in any other way. The musical expression becomes a rhythmic pattern that is consistently evolving, built on the patterns of the individual sounds that throughout maintain a consistent relationship to each other, as in Allen C. Bluedorn's definition of the process of entrainment.

For further work, this project has led to many questions that I find being interesting fields for further studies. Many of these questions concerns the conception of *rhythm* and *music*; questions such as what importance rhythm has in music, why people feel so strongly about music, why music makes people happy (or unhappy), and what it is that makes all known cultures around the world to create and listen to music.

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